

Class  $\Rightarrow$  B.Sc.(Hons.) Part I  
 Subject  $\Rightarrow$  Chemistry  
 Paper  $\Rightarrow$  IA (Physical chemistry)  
 Chapter  $\Rightarrow$  Gaseous state (Group-A)  
 Topic  $\Rightarrow$  Derivation of kinetic gas equation.

Name  $\Rightarrow$  Dr. Amarendra Kumar  
 Deptt. of chemistry  
 H.D.Jain College, ARA.

## Derivation of kinetic gas equation

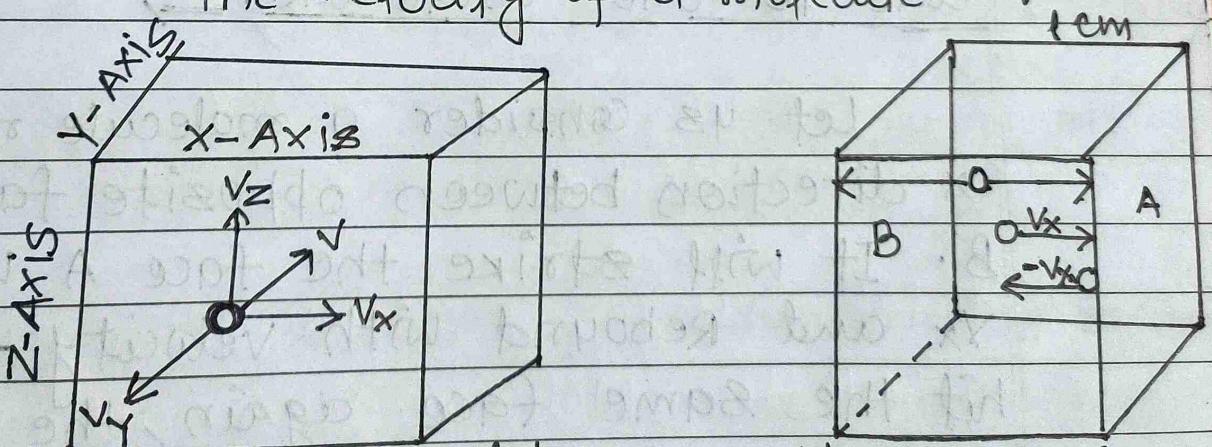
Let us consider a certain mass of gas enclosed in a cubic box at a fixed temperature.

The length of each side of the box = 1 cm

The total no. of gas molecules =  $n$

The mass of one molecule =  $m$

The velocity of a molecule =  $v$



Resolution of velocity  $v$  into components  $v_x$ ,  $v_y$  and  $v_z$ .

Molecular collisions along X-axis.

I The kinetic gas equation is derived by the following steps.

1. Resolution of velocity  $v$  of a single molecule along  $x$ ,  $y$  and  $z$  axes  $\Rightarrow$

According to the kinetic theory, a molecule of a gas moves with velocity  $v$  in any direction and resolved into the components  $v_x$ ,  $v_y$  and  $v_z$  along  $x$ ,  $y$  and  $z$  axes.

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

Now, we consider the motion of a single molecule moving with the component velocities independently in each direction.

2. The no. of collisions per second on face A due to one molecule.  $\Rightarrow$

Let us consider a molecule moving in  $x$  direction between opposite faces A and B. It will strike the face A with velocity  $v_x$  and rebound with velocity  $-v_x$ . To hit the same face again, the molecule must travel 1 cm to collide with

the opposite face B and then again l cm to return to face A. Therefore,

The time between two collisions of face A  $v_x = \frac{2l}{v_x}$  seconds

The no. of collisions per second on face A  $= \frac{v_x}{2l}$

3. The total change of momentum on all faces of the box due to one molecule only

Each impact of the molecule on the face A causes a change of momentum (mass  $\times$  velocity).

The momentum before the impact  $= mv_x$

The momentum after the impact  $= m(-v_x)$

$$\therefore \text{The change of momentum} = mv_x - (-mv_x) \\ = 2mv_x$$

But the no. of collisions per second on face A due to one molecule  $= \frac{v_x}{2l}$

$\therefore$  The total change of momentum per second on face A caused by one molecule  $=$

$$2mv_x \times \left( \frac{v_x}{2l} \right) = \frac{mv^2_x}{l}$$

The change of momentum on both the opposite faces A and B along x-axis would be double i.e.

$$2mv_x^2$$

$l$

Similarly,

The change of momentum along y axis =  $\frac{2mv_y^2}{l}$

The change of momentum along z axis =  $\frac{2mv_z^2}{l}$

∴ The overall change of momentum per second on all faces of the box will be

$$= \frac{2mv_x^2}{l} + \frac{2mv_y^2}{l} + \frac{2mv_z^2}{l}$$

$$= \frac{2m}{l} (v_x^2 + v_y^2 + v_z^2)$$

$$= \frac{2mv^2}{l} \quad \left( \because v^2 = v_x^2 + v_y^2 + v_z^2 \right)$$

4. Total change of Momentum due to impact of all the Molecules on all faces of the Box  $\Rightarrow$

Let  $N$  molecules in the box each of which is moving with a different velocity  $v_1, v_2$  and  $v_3$  respectively.

$\therefore$  The total change of momentum due to impact of all the molecules on all faces of the Box =  $\frac{2m}{l} (v_1^2 + v_2^2 + v_3^2 + \dots)$

Multiplying and dividing by  $n$  we get

$$\begin{aligned} &= \frac{2mN}{l} \left( \frac{v_1^2 + v_2^2 + v_3^2 + \dots}{n} \right) \\ &= \frac{2mN u^2}{l} \end{aligned}$$

Where  $u^2$  is the mean square velocity.

5. Calculation of pressure from change of momentum ; Derivation of kinetic gas equation  $\Rightarrow$

The change in momentum per second is called force.

$$\therefore \text{Force} = \frac{2mN u^2}{l}$$

But, Pressure =  $\frac{\text{Total force}}{\text{Total Area}}$

$$P = \frac{2mNv^2}{l} \times \frac{1}{6l^2}$$

$$= \frac{1}{3} \frac{mNv^2}{l^3}$$

Since  $l^3$  = volume of the cube  $v$

$$\therefore P = \frac{1}{3} \frac{mNv^2}{v}$$

or 
$$PV = \frac{1}{3} mNv^2$$

This is the fundamental equation of the kinetic molecular theory of gas. It is called the kinetic gas equation.